PRECALC Sec. 6.4 (B)

Ex. 1) Establish each identity:

a) $\tan(\theta + \frac{\pi}{2}) = -\cot\theta$

$$\frac{\sin(\theta + \pi/2)}{\cos(\theta + \pi/2)} = \frac{\sin\theta\cos\frac{\pi}{2} + \cos\theta \cdot \sin\frac{\pi}{2}}{\cos\theta\cos\frac{\pi}{2} - \sin\theta\sin\frac{\pi}{2}}$$

$$= \frac{0 + \cos\theta}{0 - \sin\theta} = -\frac{\cos\theta}{\sin\theta} = -\cot\theta \checkmark$$

b) $\frac{\cos(\alpha - \beta)}{\sin\alpha \sin\beta} = \cot\alpha\cot\beta + 1$

$$\frac{\cos A\cos B + \sin A\sin B}{\sin A\sin B}$$

$$= \frac{\cos A\cos B}{\sin A\sin B} + \frac{\sin A\sin B}{\sin A\sin B}$$

$$= \cot A\cot B + 1 \checkmark$$

Ex. 2) Find the EXACT value of each expression:

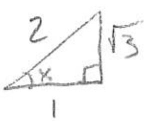
a) $\sin(\sin^{-1}\frac{1}{2} + \cos^{-1}0)$ $\Rightarrow$ $\sin(30+90) = \sin(120°) = \boxed{\frac{\sqrt{3}}{2}}$

$\sin x = \frac{1}{2}$ $\quad \cos y = 0$

$x = 30°$ $\quad y = 90°$

b) $\sin(\cos^{-1}\frac{1}{2} + \sin^{-1}\frac{3}{5}) = \sin(x+y) = \sin x\cos y + \cos x\sin y$

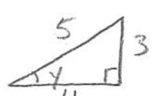
$$= \frac{\sqrt{3}}{2} \cdot \frac{4}{5} + \frac{1}{2} \cdot \frac{3}{5} = \boxed{\frac{4\sqrt{3} + 3}{10}}$$



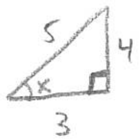
$\cos x = \frac{1}{2}$ $\quad \sin y = \frac{3}{5}$

$x = 60°$

$0 \le x \le \pi$ $\quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$



c) $\tan(\sin^{-1}\frac{4}{5} + \cos^{-1}1)$



$\sin x = \frac{4}{5}$ $\quad \cos y = 1 \Rightarrow y = 0°$

$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ $\quad 0 \le y \le \pi$

$\cos x = \frac{3}{5}$ $\quad \sin y = 0$

$$\frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x\cos y + \cos x\sin y}{\cos x\cos y - \sin x\sin y}$$

$$= \frac{\frac{4}{5}\cdot 1 + \frac{3}{5}\cdot 0}{\frac{3}{5}\cdot 1 - \frac{4}{5}\cdot 0} = \frac{\frac{4}{5}}{\frac{3}{5}} = \boxed{\frac{4}{3}}$$

Ex. 3) Write $\sin(\sin^{-1}u + \cos^{-1}v)$ as an algebraic expression containing u and v

$\sin^{-1}u = x$ $\quad \cos^{-1}v = y$ $\quad \sin(x+y) = \sin x\cos y + \cos x\sin y$

$\sin x = u$ $\quad \cos y = v$ $\quad = \boxed{uv + \sqrt{1-u^2}\cdot\sqrt{1-v^2}}$

$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ $\quad 0 \le y \le \pi$